

10.2

Arcs and Chords

5

- Goals**
- Use properties of arcs of circles.
 - Use properties of chords of circles.

VOCABULARY

Central angle

Minor arc

Major arc

Semicircle

Measure of a minor arc

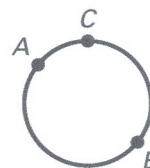
Measure of a major arc

Congruent arcs

POSTULATE 26: ARC ADDITION POSTULATE

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$$m\widehat{ABC} = \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$$



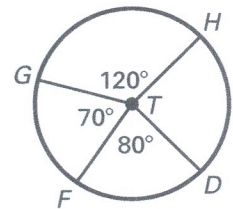
Example 1 Finding Measures of Arcs

Find the measure of each arc.

a. \widehat{DG}

b. \widehat{DGH}

c. \widehat{DH}



Solution

a. $m\widehat{DG} = m\widehat{DF} + m\widehat{FG} = \underline{\hspace{1cm}}^\circ + \underline{\hspace{1cm}}^\circ = \underline{\hspace{1cm}}^\circ$

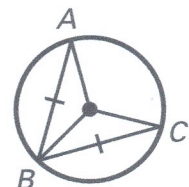
b. $m\widehat{DGH} = m\widehat{DG} + m\widehat{GH} = \underline{\hspace{1cm}}^\circ + \underline{\hspace{1cm}}^\circ = \underline{\hspace{1cm}}^\circ$

c. $m\widehat{DH} = 360^\circ - m\widehat{DGH} = 360^\circ - \underline{\hspace{1cm}}^\circ = \underline{\hspace{1cm}}^\circ$

THEOREM 10.4

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

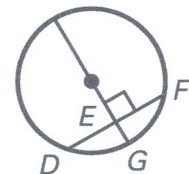
$\widehat{AB} \cong \widehat{BC}$ if and only if $\underline{\hspace{1cm}} \cong \underline{\hspace{1cm}}$.



THEOREM 10.5

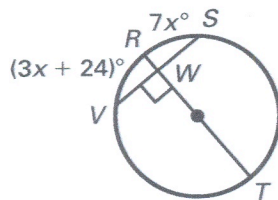
If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

$\overline{DE} \cong \overline{EF}$, $\underline{\hspace{1cm}} \cong \underline{\hspace{1cm}}$



✓ Checkpoint Complete the follo

1. Use Theorem 10.5 to find $m\widehat{RS}$.



Example 2 Using Theorem 10.5

Find $m\widehat{KM}$ using Theorem 10.5.

$m\widehat{KM} = m\widehat{MN}$

$5x^\circ = (7x - 16)^\circ$

$\underline{\hspace{1cm}} = \underline{\hspace{1cm}}x - 16$

$\underline{\hspace{1cm}} = \underline{\hspace{1cm}}x$

$\underline{\hspace{1cm}} = x$

Answer $m\widehat{KM} = 5x^\circ = \underline{\hspace{1cm}}^\circ = \underline{\hspace{1cm}}^\circ$.

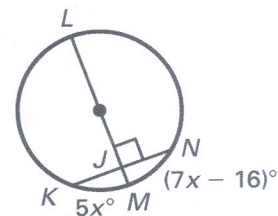
Theorem 10.5

Substitute.

Subtract $\underline{\hspace{1cm}}$ from each side.

Add $\underline{\hspace{1cm}}$ to each side.

Divide.

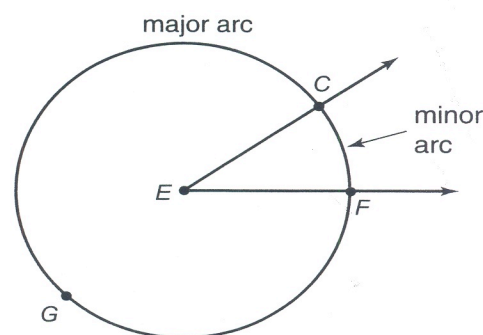


Study Guide

10.2 Blue

Arcs and Central Angles

An angle whose vertex is at the center of a circle is called a **central angle**. A central angle separates a circle into two arcs called a **major arc** and a **minor arc**. In the circle at the right, $\angle CEF$ is a central angle. Points C and F and all points of the circle interior to $\angle CEF$ form a minor arc called arc CF . This is written \widehat{CF} . Points C and F and all points of the circle exterior to $\angle CEF$ form a major arc called \widehat{CGF} .



You can use central angles to find the degree measure of an arc. The arcs determined by a diameter are called **semicircles** and have measures of 180.

Examples: In $\odot R$, $m\angle ARB = 42$ and \widehat{AC} is a diameter.

- 1 Find $m\widehat{AB}$.

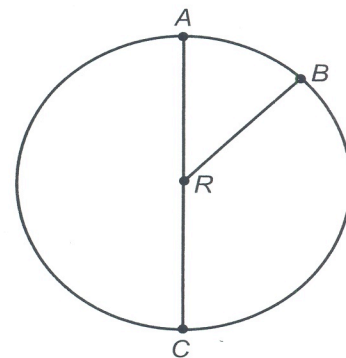
Since $\angle ARB$ is a central angle and $m\angle ARB = 42$, then $m\widehat{AB} = 42$.

- 2 Find $m\widehat{ACB}$.

$$m\widehat{ACB} = 360 - m\angle ARB = 360 - 42 \text{ or } 318$$

- 3 Find $m\widehat{CAB}$.

$$\begin{aligned} m\widehat{CAB} &= m\widehat{ABC} + m\widehat{AB} \\ &= 180 + 42 \\ &= 222 \end{aligned}$$



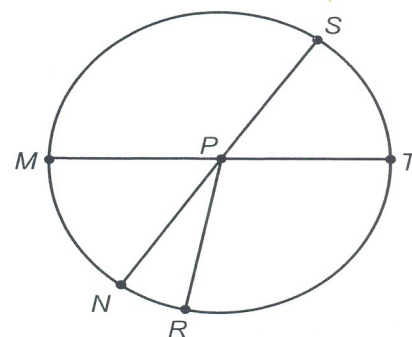
Refer to $\odot P$ for Exercises 1–4. If \widehat{SN} and \widehat{MT} are diameters with $m\angle SPT = 51$ and $m\angle NPR = 29$, determine whether each arc is a minor arc, a major arc, or a semicircle. Then find the degree measure of each arc.

1. $m\widehat{NR}$

2. $m\widehat{ST}$

3. $m\widehat{TSR}$

4. $m\widehat{MST}$



Study Guide

10.2 Blue

Arcs and Chords

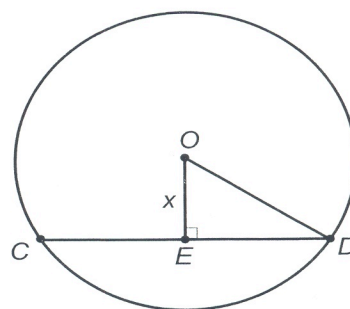
The following theorems state relationships between arcs, chords, and diameters.

- In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.
- In a circle, a diameter bisects a chord and its arc if and only if it is perpendicular to the chord.

Example: In the circle, O is the center, $OD = 15$, and $CD = 24$. Find x .

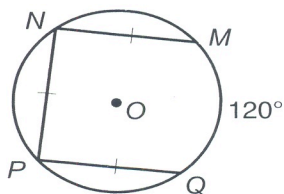
$$\begin{aligned} ED &= \frac{1}{2} CD \\ &= \frac{1}{2} (24) \\ &= 12 \end{aligned}$$

$$\begin{aligned} (OE)^2 + (ED)^2 &= (OD)^2 \\ x^2 + 12^2 &= 15^2 \\ x^2 + 144 &= 225 \\ x^2 &= 81 \\ x &= 9 \end{aligned}$$

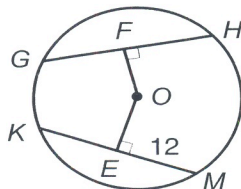


In each circle, O is the center. Find each measure.

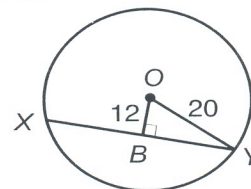
1. $m\widehat{NP}$



2. KM



3. XY



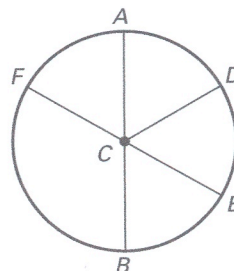
- Suppose a chord is 20 inches long and is 24 inches from the center of the circle. Find the length of the radius.
- Suppose a chord of a circle is 5 inches from the center and is 24 inches long. Find the length of the radius.
- Suppose the diameter of a circle is 30 centimeters long and a chord is 24 centimeters long. Find the distance between the chord and the center of the circle.

Practice A

For use with pages 603–611

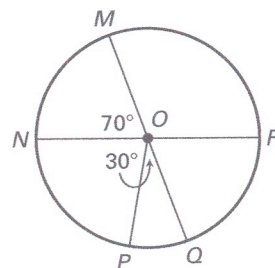
Determine whether the arc is a *minor arc*, a *major arc*, or a *semicircle* of $\odot C$.

1. \widehat{AE}
2. \widehat{AEB}
3. \widehat{FDE}
4. \widehat{DFB}
5. \widehat{FA}
6. \widehat{BE}
7. \widehat{BDA}
8. \widehat{FB}



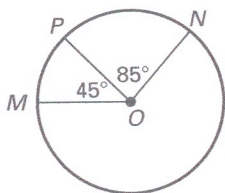
\overline{MQ} and \overline{NR} are diameters. Find the indicated measure.

9. $m\widehat{MN}$
10. $m\widehat{NQ}$
11. $m\widehat{NQR}$
12. $m\widehat{MRP}$
13. $m\widehat{QR}$
14. $m\widehat{MR}$
15. $m\widehat{QMR}$
16. $m\widehat{PQ}$
17. $m\widehat{PRN}$
18. $m\widehat{MQN}$

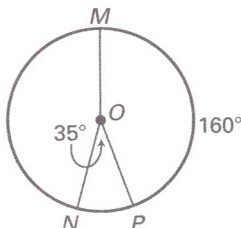


Find the measure of \widehat{MN} .

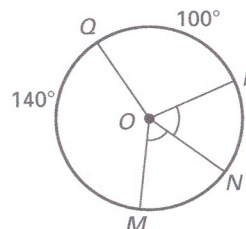
19.



20.

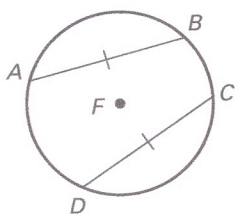


21.

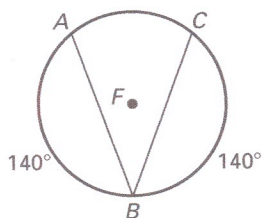


What can you conclude about the diagram? State a postulate or theorem that justifies your answer.

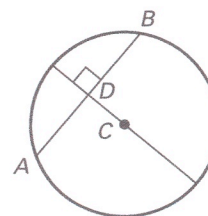
22.



23.

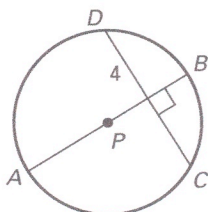


24.

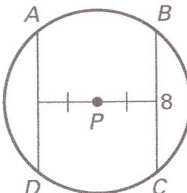


Find the indicated measure for $\odot P$.

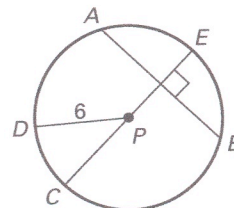
25. $DC = \underline{\hspace{1cm}} ?$



26. $AD = \underline{\hspace{1cm}} ?$



27. $EC = \underline{\hspace{1cm}} ?$



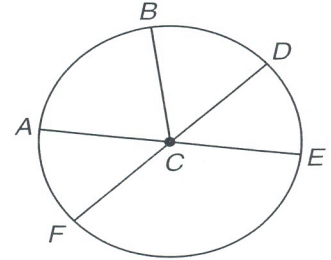
Skills Practice

10.2 Blue

Arcs and Central Angles

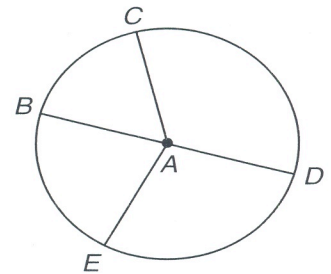
Find each measure in $\odot C$ if $m\angle ACB = 80$, $m\widehat{AF} = 45$, and \overline{AE} and \overline{FD} are diameters.

1. $m\angle ACF$
2. $m\widehat{AB}$
3. $m\angle FCE$
4. $m\widehat{EF}$
5. $m\widehat{ABE}$
6. $m\angle BCE$
7. $m\widehat{AFE}$
8. $m\angle DCE$
9. $m\widehat{DE}$
10. $m\angle BCD$
11. $m\widehat{BAE}$
12. $m\widehat{ABF}$



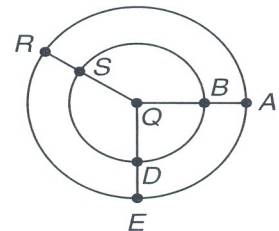
In $\odot A$, \overline{BD} is a diameter, $m\angle BAE = 85$, and $m\angle CAD = 120$. Determine whether each statement is true or false.

13. $m\angle BAC = 60$
14. $m\widehat{CD} = m\angle CAD$
15. $\angle ABE$ is a central angle.
16. $m\angle BAC = m\angle DAE$
17. $m\widehat{CED} = 220$
18. $m\widehat{BCD} = 180$
19. $m\widehat{CE} = 145$
20. $m\angle DAE = m\widehat{DE}$



Q is the center of two circles with radii \overline{QD} and \overline{QE} . If $m\angle AQE = 90$ and $m\widehat{RE} = 115$, find each measure.

21. $m\widehat{AE}$
22. $m\angle RQE$
23. $m\widehat{AR}$
24. $m\angle RQA$
25. $m\widehat{AER}$
26. $m\widehat{BSD}$
27. $m\widehat{DS}$
28. $m\widehat{BD}$

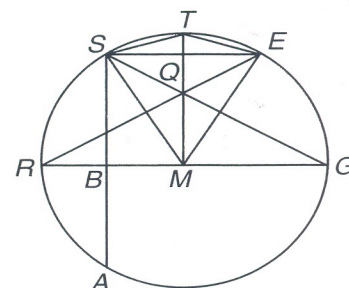


Skills Practice *10, 2 Blue*

Arcs and Chords

Complete each sentence.

- If $\widehat{SG} \cong \widehat{RE}$, then $\overline{SG} \cong$ _____?
- If $\widehat{ST} \cong \widehat{ET}$, then $\triangle SMT \cong \triangle$ _____?
- If $\overline{TM} \perp \overline{RG}$, then $\widehat{RT} \cong$ _____?
- If $\widehat{ST} \cong \widehat{TE}$, $\overline{ST} \cong$ _____?
- If $\overline{RG} \perp \overline{AS}$, then $\overline{SB} \cong$ _____?
- If $\widehat{RE} \cong \widehat{SG}$, then $\triangle RME \cong \triangle$ _____?
- If $\widehat{RE} \cong \widehat{SG}$, then $\widehat{RE} \cong$ _____?
- If $\overline{TM} \perp \overline{RG}$, then $\widehat{ST} \cong$ _____?
- If $\overline{TM} \perp \overline{RG}$, then $\overline{SQ} \cong$ _____?
- If $\overline{TM} \perp \overline{RG}$ and $\widehat{ST} \cong \widehat{TE}$, then $\triangle SQT \cong \triangle$ _____?
- If $\overline{SQ} \cong \overline{EQ}$, then $\overline{TM} \perp$ _____?
- If $\widehat{SR} \cong \widehat{AR}$, then $\overline{RG} \perp$ _____?



Use $\odot B$, where $\overline{BX} \perp \overline{WY}$, to complete each sentence.

- If $BW = 23$, then $BY =$ _____?
- If $WY = 38$, then $WZ =$ _____?
- If $WZ = 15$, then $WY =$ _____?
- If $BZ = 6$ and $WZ = 8$, then $WB =$ _____?
- If $WB = 15$ and $BZ = 9$, then $WZ =$ _____?
- If $WY = 40$ and $BZ = 15$, then $WB =$ _____?
- If $BY = 30$ and $BZ = 18$, then $WY =$ _____?
- If $m\widehat{WY} = 110$, then $m\widehat{WX} =$ _____?

